



# Efficient Bijective Parameterizations

Su J P, Ye C, Liu L, et al. Efficient bijective parameterizations[J]. ACM Transactions on Graphics (TOG), 2020, 39(4): 111: 1-111: 8.



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- 02 参数化
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# 01

## PART

# 图形学是什么

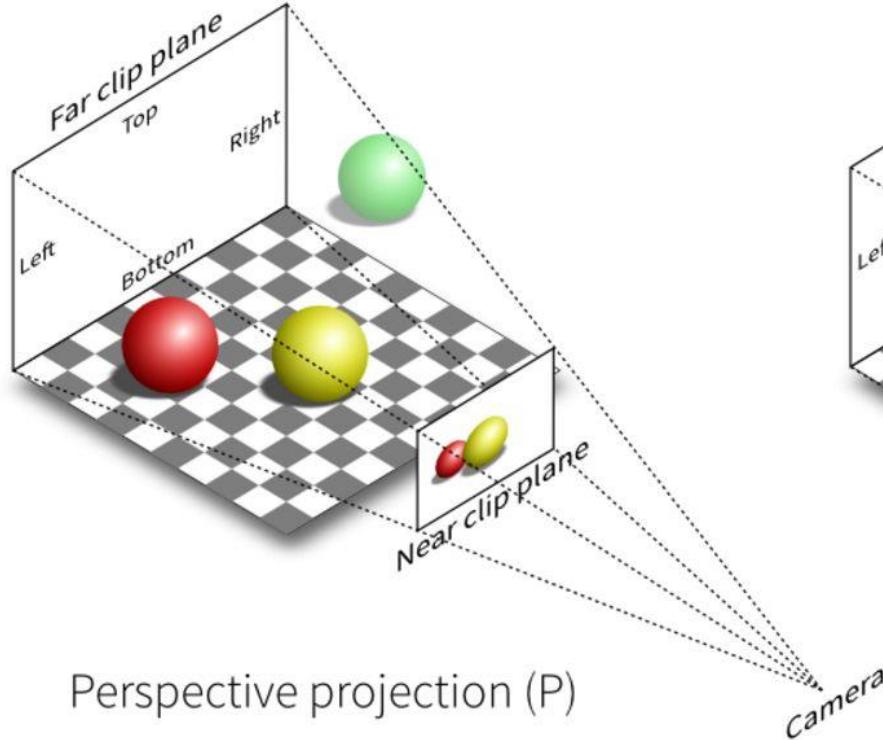


## 图形学是什么

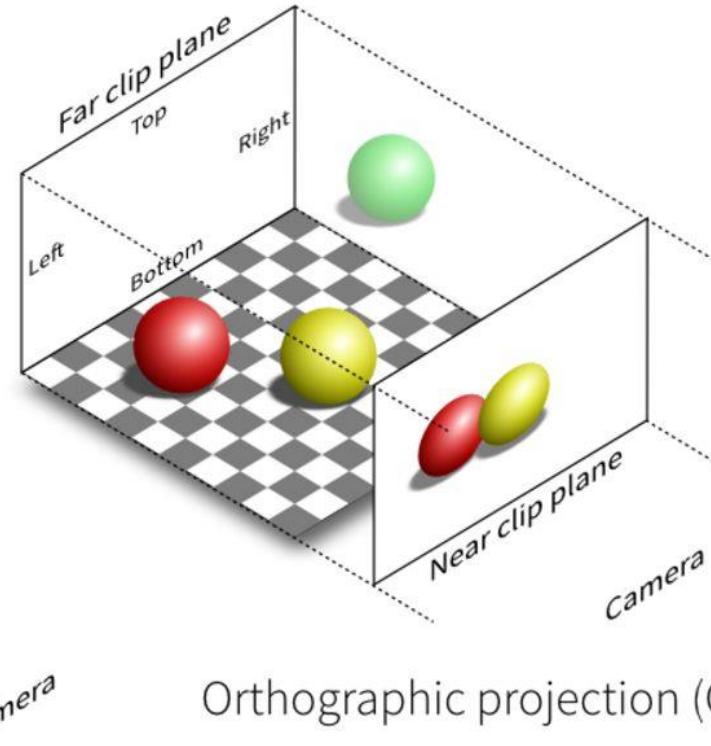
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- ★ 计算机图形学可以说是一门用计算机生成图形图像的学科。
- ★ 传统图形学致力于生成可被传统显示器显示的二维图像，也就是三维世界的二维投影。
- ★ 需要构建三维世界的表示方法以及其相关操作，这需要基础的立体几何、投影几何知识。

# 图形学是什么



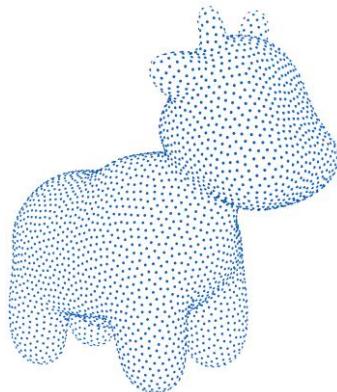
Perspective projection (P)



Orthographic projection (O)

# 图形学是什么

于是就有了传统计算机图形学的三大研究方向：



几何



渲染

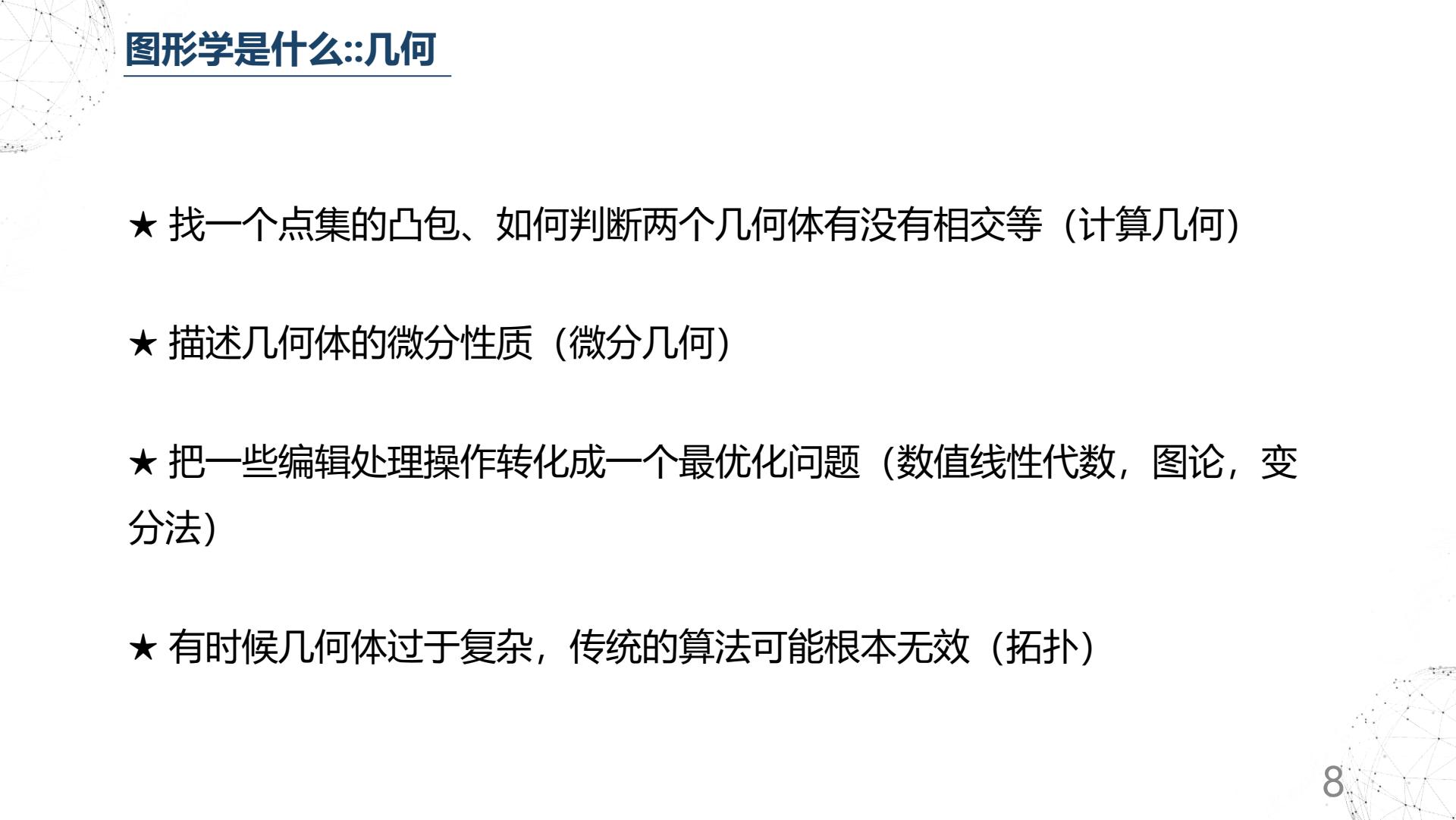


动画



# 图形学是什么::几何

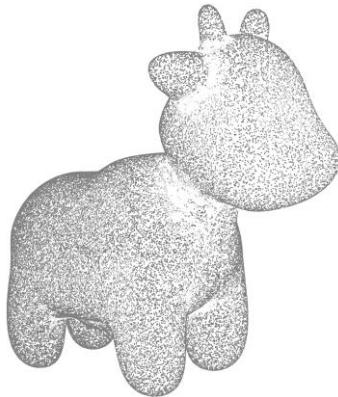
- ★ 物体是什么形状?
- ★ 如何既省空间又精确地表示这个形状?
- ★ 如何从零起步为我心中所想的一个形状建模?
- ★ 如何编辑现有的形状让他符合我的要求与目的?



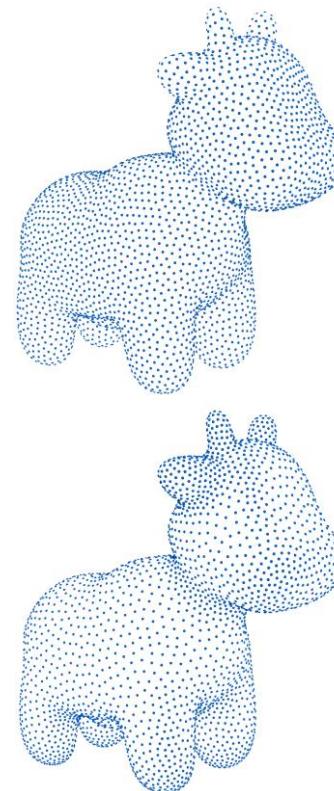
## 图形学是什么::几何

- ★ 找一个点集的凸包、如何判断两个几何体有没有相交等（计算几何）
- ★ 描述几何体的微分性质（微分几何）
- ★ 把一些编辑处理操作转化成一个最优化问题（数值线性代数，图论，变分法）
- ★ 有时候几何体过于复杂，传统的算法可能根本无效（拓扑）

# 图形学是什么::几何

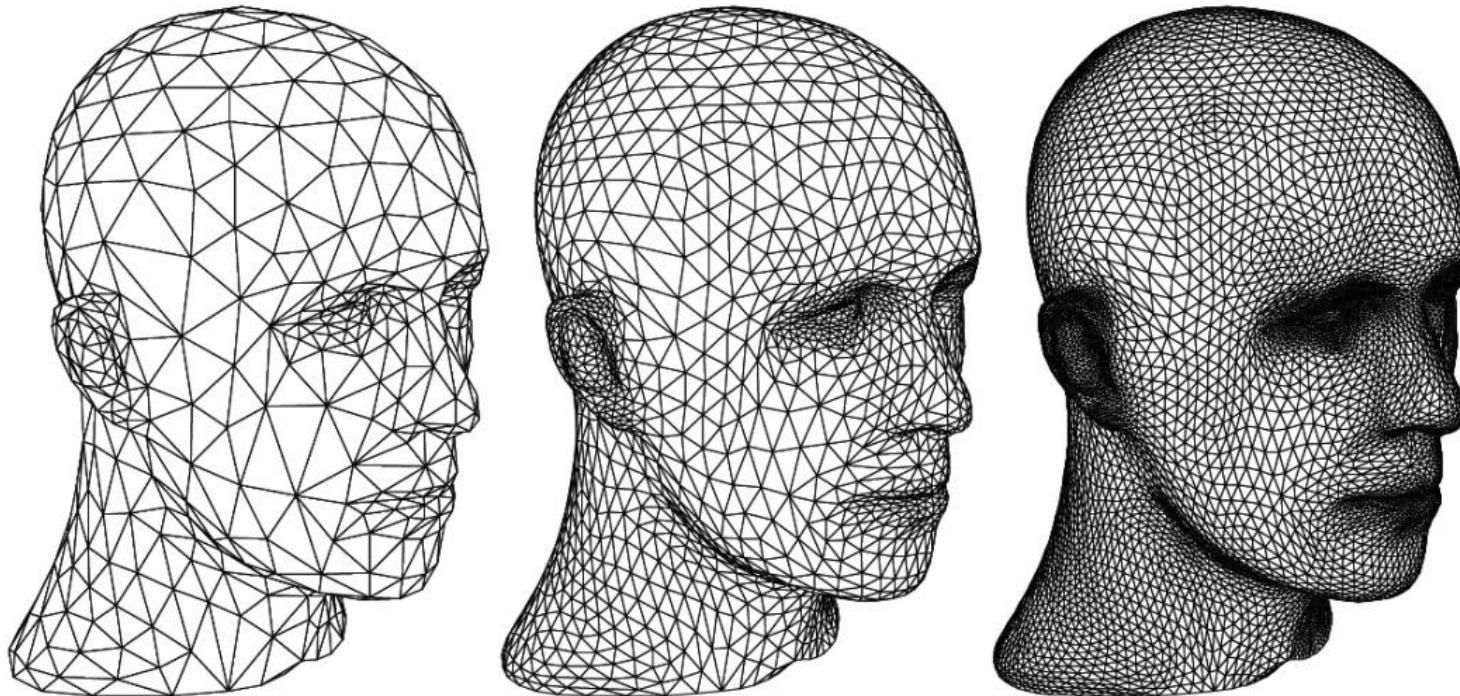


均匀重采样  
带密度重采样



重采样 5k 个点

# 图形学是什么::几何

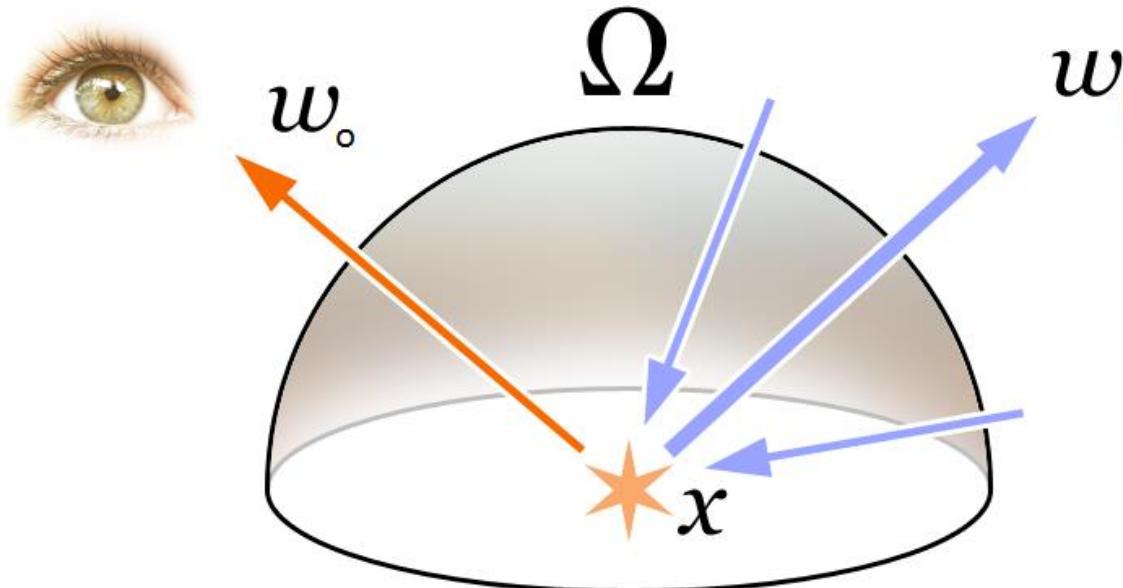


# 图形学是什么::渲染



## 图形学是什么::渲染

$$L_0(x, \omega_0) = L_e(x, \omega_0) + \int_{\Omega} f_r(x, \omega_i, \omega_0) L_i(x, \omega_i) (\omega_i \cdot \vec{n}) d\omega_i$$



Kajiya J T. The rendering equation[C]//Proceedings of the 13th annual conference on Computer graphics and interactive techniques. 1986: 143-150.

- ★ 可以把物体大致分为三类：固体、角色、流体（无固定形态）
- ★ 为了让动画看起来真实，你需要用力学和运动学知识去描述、规范他们的运动。
- ★ 对于角色，控制理论（所以也就还包括随机过程）、生物学也是需要的。
- ★ 流体的话，那就是流体力学了（所以也就还包括微分方程和数值分析）。



## 图形学是什么::其它

- ★ 图像的拍摄和显示技术，也就是**计算成像**，这又需要物理光学的知识。
- ★ 再比如**3D打印**，不再局限于生成图像，也能生成实体，这又需要一些材料科学的知识。
- ★ 以及最近炒得火热的**增强现实(AR)**，要想增强现实，首先理解现实。所以需要计算机视觉。
- ★ 另外，虽说目前很火的机器学习是解决**逆问题**的有利工具，不过其在图形学这个主要以数学建模驱动来解决正向问题的领域中也偶尔会被用到。



## 02 PART

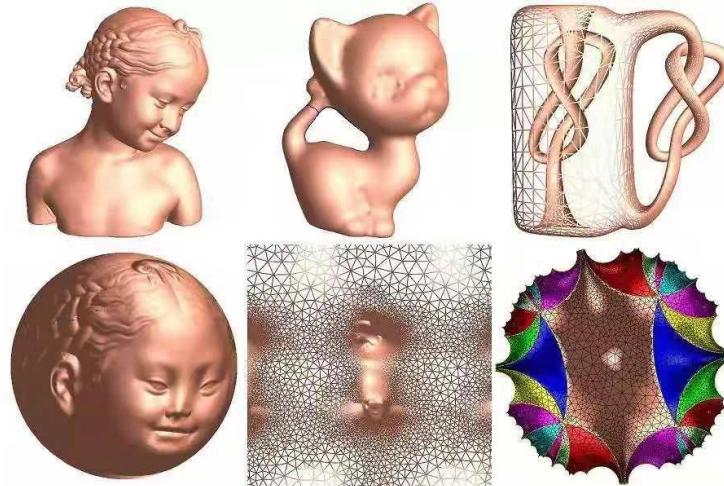
# 参数化

## 参数化

★ 问题：给定一张曲面，如何求其二维的参数定义域？

★ 又称为：曲面展开。

找到一个映射  $f: S \subset R^3 \rightarrow \Omega \subset R^2$



A faint, abstract background consisting of numerous small, dark gray dots connected by thin, light gray lines, forming a complex network or mesh pattern.

**03**  
PART

# **Efficient Bijective Parameterizations**

## Prof. Dr. Ligang Liu (刘利刚)

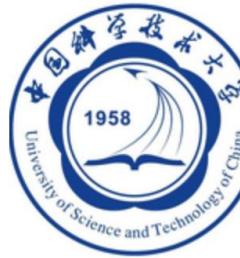


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# Effective Bijective Parameterizations::作者介绍

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## Research Interests [Top]

My research interest is computer graphics. More specifically:

- **Geometry modeling and processing**
  - Surface reconstruction
  - Shape modeling and creation
  - Surface parameterization
  - Shape editing and morphing
- **Geometric optimization in 3D printing (Computational Fabrication)**
  - Geometric modeling in 3D printing
  - Structure analysis and optimization
  - Applications using 3D printing
- **Sparse learning and deep learning in geometry processing**
  - Sparse representation
  - Low rank
  - Geometric deep learning
- **Shape analysis (high-level geometry processing)**
  - Sketch-based interactive shape segmentation
  - Symmetry analysis
  - Feature detection and analysis for 3D shapes
  - Co-segmentation of shape set
- **Image and video processing**
  - Geometry-driven image manipulation: image retargeting, human reshaping
  - Computational aesthetics: photo composition, black-and-white photos
  - Video editing

# Effective Bijective Parameterizations::符号说明

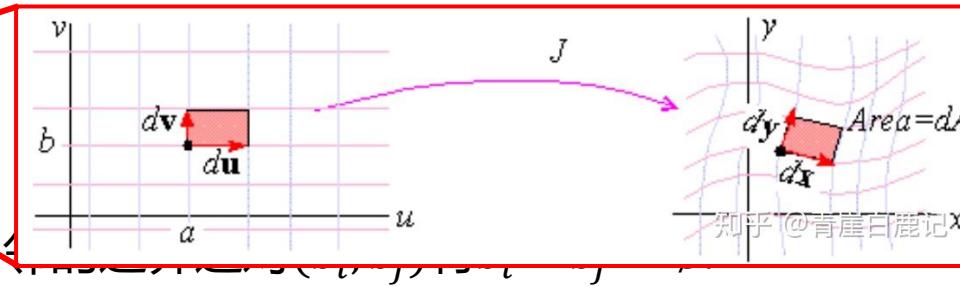
符号	解释
$\mathcal{M}$	输入网格
$\mathcal{F} = \{f_i, i = 1, 2, \dots, \mathcal{N}_f\}$	三角形集合
$\mathcal{V} = \{v_i, i = 1, 2, \dots, \mathcal{N}_v\}$	点集
$\widehat{\mathcal{M}}$	参数化网格
$\widehat{\mathcal{F}}$	参数化三角形集合
$\widehat{\mathcal{V}}$	参数化点集
$J_i$	从 $f_i$ 到 $\widehat{f}_i$ 的仿射变换对应的 2×2的雅可比矩阵
$\mathcal{B}$	$\widehat{\mathcal{M}}$ 的边界
$\mathcal{E}_b = \{b_i\}$	边界边集合
$\mathcal{V}_b = \{x_i\}$	边界点集合

# Effective Bijective Parameterizations::问题描述

**问题描述** 给定  $\mathcal{M}$ , 我们希望计算一个具有低失真率的双射参数化网格  $\hat{\mathcal{M}}$ ,  
且它还满足下述两个约束:

- ★  $\det J_i > 0$ ;

- ★ 对于任意不相交的  $i, j$ ,  $J_i \neq J_j$ .





# Effective Bijective Parameterizations::问题描述

我们可以将其转化为无约束的最优化问题：

$$\min_{\hat{\mathcal{M}}} g(\hat{\mathcal{M}}) = E_d(\hat{\mathcal{M}}, \mathcal{M}) + \lambda_b E_b(\mathcal{B}), \quad (*)$$

$E_d(\hat{\mathcal{M}}, \mathcal{M})$	衡量三角形的失真程度
$E_b(\mathcal{B})$	边界障碍函数
$\lambda_b > 0$	权重

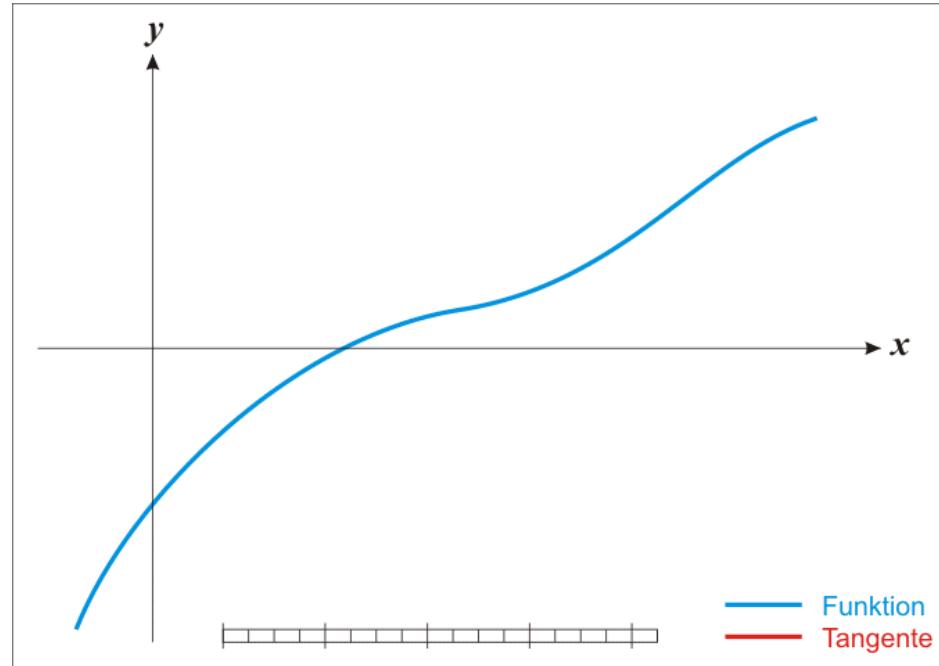
# Effective Bijective Parameterizations::牛顿法

$$x = x - \frac{f(x)}{f'(x)}$$

↓

$$\hat{M} = \hat{M} - \frac{g'(\hat{M})}{g''(\hat{M})}$$

*g的Hessian矩阵H*



# Effective Bijective Parameterizations::优化

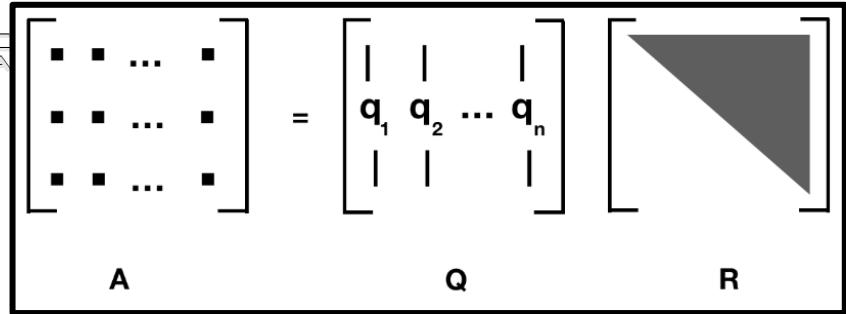
$$\frac{g'(\hat{\mathcal{M}})}{g''(\hat{\mathcal{M}})} = ? \rightarrow g''(\hat{\mathcal{M}})X = g'(\hat{\mathcal{M}}) \rightarrow HX = b$$

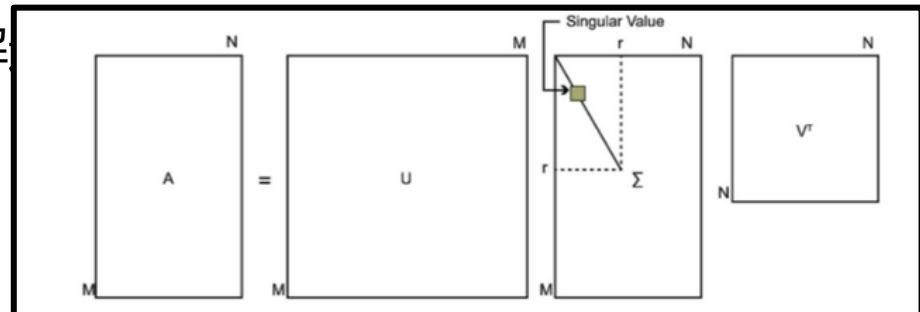
几乎所有解决稀疏线性系统的算法

★ 分析  $H$  的非零结构；

★ 对  $H$  进行数值上的矩阵分解；

★ 利用上述阶段得到的矩阵分解

$$A = Q R$$


$$A = U \Sigma V^T$$


稀疏的Hessian矩阵 $H$ 很难满足如下两个约束：

- ★  $H$ 的非零结构是固定的；
- ★  $H$ 的非零元素需要尽可能的少。

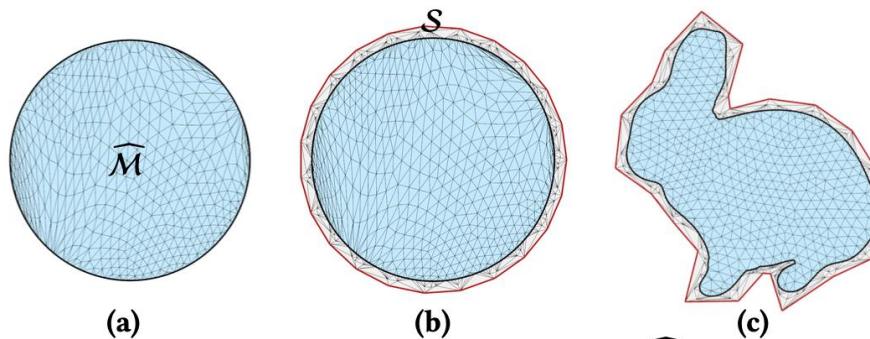


Fig. 3. We expand the initial parameterization mesh  $\widehat{\mathcal{M}}$  (a) (in blue) with a tight shell mesh  $\mathcal{S}$  (b) (in gray). The shell  $\mathcal{S}$  conforms to the boundary of  $\widehat{\mathcal{M}}$  and contains much smaller number of boundary edges. We perform the optimization of Eq. (6) on the expanded mesh  $\widehat{\mathcal{M}} \cup \mathcal{S}$  and obtain a bijective parameterization result (c).

[Tutte, 63],  
[Maxwell, 1864]

# Effective Bijective Parameterizations::实验结果

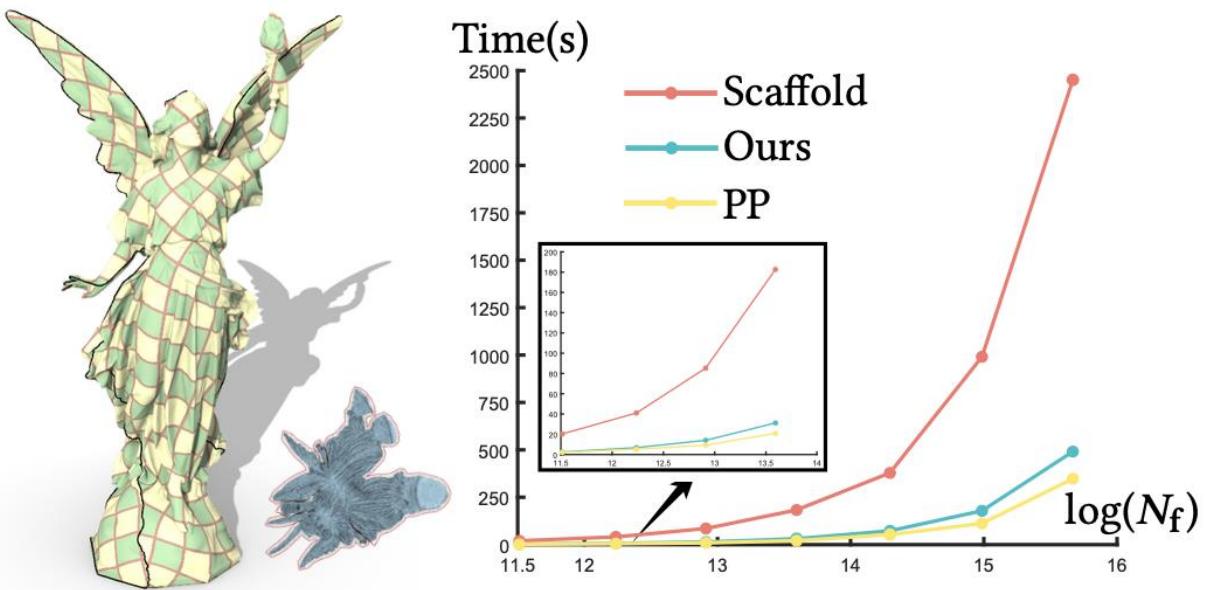


Fig. 8. Scalability test for three approaches including Scaffold, PP, and our method. A Lucy surface is represented by the meshes with increasing resolutions (100000, 206215, 405340, 800413, 1615932, 3223846, 6376340) is employed. The left graph shows the 100K Lucy model parameterized by our algorithm. The right graph plots the running time (in seconds) vs.  $\log(N_f)$ .

# Effective Bijective Parameterizations::实验结果



(a) (1.041, 24) (b) (1.066, 31) (c) (1.049, 26) (d) (1.044, 30) (e) (1.107, 35)

Fig. 9. Our method is robust to different triangulations such as (a) Original, (b) Isotropic, (c) Anisotropic, (d) Nonuniform, (e) Noisy. The text below each model indicates the resulting  $E_d(\widehat{\mathcal{M}}, \mathcal{M})$  and the iteration number. We use the LCT method [Fu et al. 2014] to generate the anisotropic mesh.

# Effective Bijective Parameterizations::实验结果

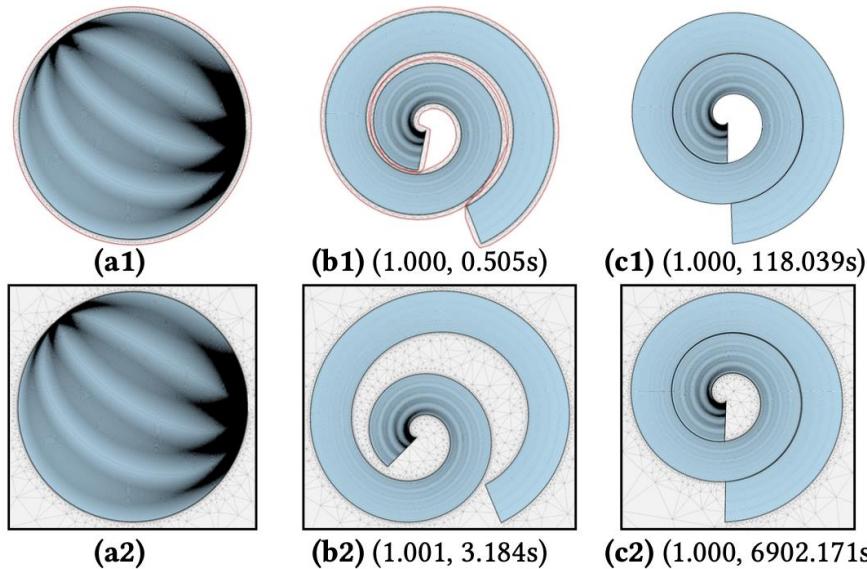


Fig. 12. A bijective map from a circle (left) to a spiral (right). Our algorithm first augments the circle with a shell mesh (a1), then we achieve the result (b1) by our method (i.e., the method in Fig. 2 (c)), and finally remove the shell mesh and obtain the result (c1) using the method in Fig. 2 (b). Scaffold first augments the circle with a scaffold mesh (a2), then obtains the convergence result (b2). If Scaffold is not limited by the convergence condition, it can obtain a result (c2) comparable to ours (c1). The text below each image indicates the resulting  $E_d(\widehat{\mathcal{M}}, \mathcal{M})$  and the running time in seconds.

# Effective Bijective Parameterizations::实验结果

	QN	Sca	PP	Ours
Order	1 <sup>st</sup> +	2 <sup>nd</sup>	2 <sup>nd</sup>	2 <sup>nd</sup>
Symbolic	-	✗	✓	✓
Bijection	✓	✓	✗	✓

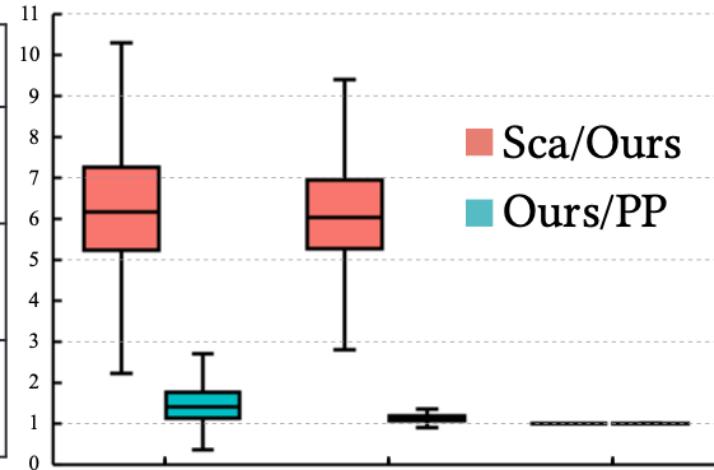
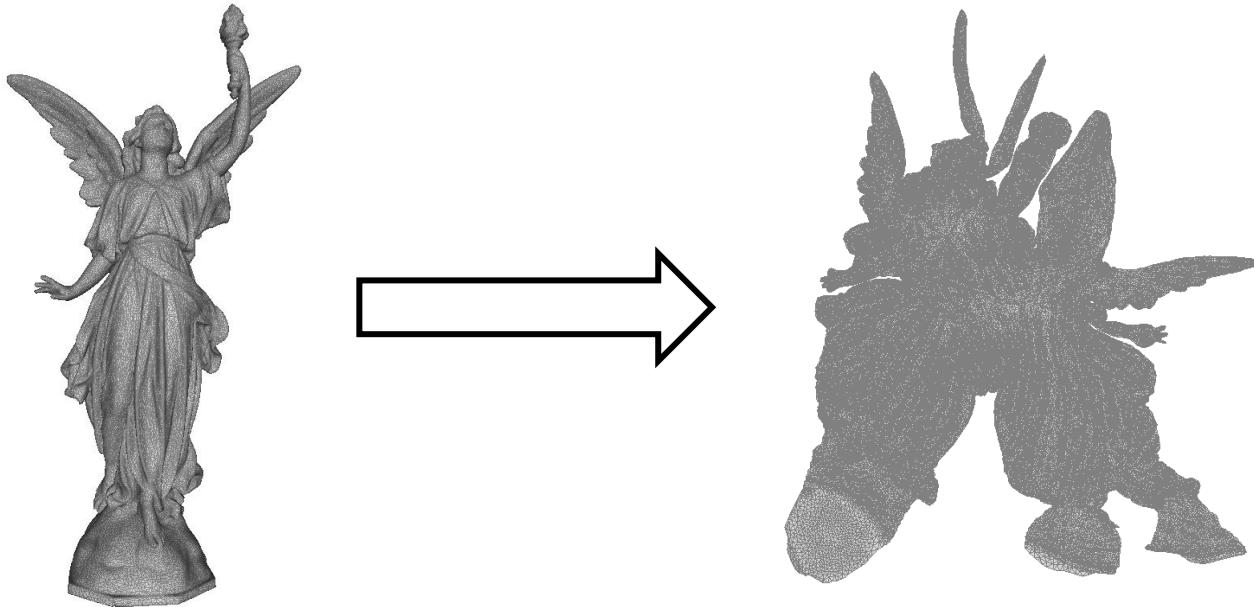


Fig. 15. Efficiency comparison summary. Left table shows convergence order, symbolic prefactorization, and bijection comparisons for QN, Scaffold, PP and Ours. Right graph shows statistics by box-plots. Left: running time ratios  $T_{\text{Sca}}/T_{\text{Ours}}$  and  $T_{\text{Ours}}/T_{\text{PP}}$ . Middle: average per iteration time ratios  $T_{\text{Sca}}^{\text{avg}}/T_{\text{Ours}}^{\text{avg}}$  and  $T_{\text{Ours}}^{\text{avg}}/T_{\text{PP}}^{\text{avg}}$ . Right: energy ratios  $E_{\text{Sca}}/E_{\text{Ours}}$  and  $E_{\text{Ours}}/E_{\text{PP}}$ .

# Effective Bijective Parameterizations::实验结果

```
Windows PowerShell  
PS D:\PartitionF\Documents\技术分享\20210104\exe> .\shell.exe .\input_mesh.obj  
.input_mesh bijective parameterization begin ...  
COMP ===== time_consumption: 2.11; mesh energy: 1.04688; sum_iter: 27  
.input_mesh bijective parameterization finish !  
请按任意键继续. . .  
PS D:\PartitionF\Documents\技术分享\20210104\exe>
```





# Effective Bijective Parameterizations::总结

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- ★ 将参数化问题转化为最优化问题，并使用牛顿法进行求解。
- ★ 提出shell mesh以固定Hessian矩阵的非零结构，并避免求解系数矩阵具有大量非零元素的线性系统，从而实现快速优化。

# THANKS

